Threshold Synthesis Problem

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Why did I come?

I would like to present my work on the threshold synthesis problem because I would like to know:

- Has the project Boole worked on it?
- Has the project Boole worked on related problems?
- Are you aware of other recent works in this area?

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Definition: A linear pseudo-Boolean constraint is

$$a_1 l_1 + \ldots + a_n l_n \geq d \qquad a_i, d \in \mathbb{N}, l_i \in \{x_i, \bar{x}_i \equiv 1 - x_i\}.$$

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Threshold Synthesis Problem: given a Boolean function (in DNF), is it a threshold function?

Example

$2x_1 + x_2 + x_3 \geq 2$ represents the function $x_1 \vee (x_2 \wedge x_3)$.

Algorithm

Given

$$\phi \equiv \begin{array}{l}(x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (x_1 \wedge x_4) \vee (x_1 \wedge x_5) \vee \\ (x_2 \wedge x_3) \vee (x_2 \wedge x_4) \vee (x_3 \wedge x_4 \wedge x_5).\end{array}$$

ϕ	$(x_2 \wedge x_3)\vee$ $(x_2 \wedge x_4)\vee$	$x_3 \wedge x_4 \wedge x_5$	*false* $x_4 \wedge x_5$	*false* *false* x_5	*false* *false* *false* *true*
		$x_3 \vee x_4$	x_4 *true*	*false* *true* *true*	
	$x_2 \vee x_3 \vee x_4 \vee x_5$	$x_3 \vee x_4 \vee x_5$ *true*	$x_4 \vee x_5$ *true* *true*	x_5 *true* *true* *true*	*false* *true* *true* *true* *true*

$4x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq \dots$	$3x_2 + 2x_3 + 2x_4 + x_5 \geq \dots$	$2x_3 + 2x_4 + x_5 \geq \dots$	$2x_4 + x_5 \geq \dots$	$x_5 \geq \dots$	$\sum_{i=6}^{5} a_i x_i \geq \dots$
$(4, 5]$	$(4, 5]$	$(4, 5]$	$(3, \infty]$	$(1, \infty]$	$(0, \infty]$
			$(2, 3]$	$(1, \infty]$	$(0, \infty]$
				$(0, 1]$	$(0, \infty]$
					$(-\infty, 0]$
		$(1, 2]$	$(1, 2]$	$(1, \infty]$	
			$(-\infty, 0]$	$(-\infty, 0]$	
				$(-\infty, 0]$	
	$(0, 1]$	$(0, 1]$	$(0, 1]$	$(0, 1]$	$(0, \infty]$
			$(-\infty, 0]$	$(-\infty, 0]$	$(-\infty, 0]$
		$(-\infty, 0]$	$(-\infty, 0]$	$(-\infty, 0]$	$(-\infty, 0]$
				$(-\infty, 0]$	$(-\infty, 0]$
					$(-\infty, 0]$

Questions?

- Has the project Boole worked on it?
- Has the project Boole worked on related problems, such as recognition of regular or shellable functions?
- Are you aware of other recent works in this area?